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Department of AERONAUTICS and ASTRONAUTICS
STANFORD UNIVERSITY

Tenth Semiannual Status Report

November, 1968

ON THE ENGINEERING PORTION OF A RESEARCH PROGRAM

TO PERFORM A GYRO TEST OF GENERAL RELATIVITY IN A SATELLITE AND DEVELOP ASSOCIATED CONTROL TECHNOLOGY

CASE FILE COPY

at

Stanford University

under

Research Grant NsG-582
National Aeronautics and Space Administration

(Principal investigators for the engineering portion of the program are Professors Robert H. Cannon, Jr., Benjamin O. Lange, and Dr. Daniel B. DeBra.)

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I. INTRODUCTION

Stanford University is engaged in a program to perform the Schiff Gyro Test of General Relativity in a satellite, and to develop control technology associated with doing so. The program was conceived by Stanford in 1961 and is described in detail in a proposal for support [Ref. 1] submitted to NASA in November, 1962, in Engineering Status Report Nos. 1 to 9 [Refs. 2 through 10], and in companion Status Reports from the Physics Department.

On the basis of Ref. 1, a Grant -- NsG-582 -- was awarded to Stanford by NASA on 8 May 1964 with a retroactive starting date of 1 October 1963. The present report describes research performed in the Department of Aeronautics and Astronautics during the tenth half-year of the NASA grant period from May, 1968, through October, 1968, and discusses present status of the program.

The cooperative engineering effort with the Stanford Physics Department has continued to concentrate on the engineering design aspects of the gyro and telescope readouts, and the conceptual development of the electronics instrumentation and attitude control systems. The telescope simulator at Davidson's is currently being modified to give improved light transmission, following the promising results already described in the Ninth Semi-Annual Report (Physics Portion).

The attitude control studies have led to an extremely interesting and promising new concept for improving the pointing accuracy of the gyro-telescope structure by means of an inner servo-loop driven by a superconducting actuator within the cryogenic environment. Important progress has also been made in the thruster mechanization for the attitude control jets, and a test chamber has been designed and constructed for evaluating valve performance in the laboratory.

We have felt strongly from the beginning that to be successful in an orbital experiment as difficult as the Gyro Test of General Relativity, it is essential that we achieve engineering experience with simpler satellite systems. It is important to do so in a way that will both contribute directly to the objectives of the General Relativity experiment and also, if possible, produce scientific and technological results of importance in their own right while being relatively inexpensive. Two such preliminary satellite experiments have therefore been conceived and pursued strongly at Stanford University, and at this time we believe important steps have been made toward accomplishing both:

- One is a piggy-back flight of a cryogenic dewar, intended to demonstrate for the first time the feasibility of maintaining superfluid helium in space. It will permit evaluation of the thermal design in a zero-g environment and will afford us considerable experience in the instrumentation of a payload. Several of the concepts utilized in the space dewar, including the use of multiple heat exchanges and a high-conductivity superfluid plug, have already been designed into the helium dewar for the laboratory experiment which is presently under construction by AGS, Inc., Waltham, Massachusetts.

- The other preliminary experiment is designed to prove out in orbit the concept of a zero-g gyro environment, as proposed in Ref. 1, to provide experience in a satellite flight program, and at the same time to produce important scientific data. Two types of data may be collected -- aeronomy and geodesy. The aeronomy data will allow improvement in the dynamic modeling of the earth's atmosphere. The geodesy data will allow determination of the Love numbers which describe the response of the earth to tidal forces and of the higher zonal harmonics in a spherical harmonic expansion of the earth's field. This experiment, called a Drag-Free Satellite, has been carefully analyzed and demonstrated in laboratory physical simulation. The dynamics, control, and uses of the Drag-Free Satellite and of unsupported gyroscopes, along with a trajectory error analysis, are described in Refs. 11, 12 and 13.

Planning with the Marshall Space Flight Center for the dewar flight is proceeding well.

Extensive design study and simulation at Stanford for both aeronomy and geodesy missions have led to a sequence of flight proposals. A proposal [Ref. 14] to fly a Drag-Free Satellite in a new aeronomy mission and an addendum [Ref. 15] to the flight proposal were submitted to NASA, and two preliminary presentations were made before the Planetary Atmospheres Subcommittee of the NASA Space Science Steering Committee. The mission would have exploited the ability of the Drag-Free Satellite in a unique way to obtain atmospheric density data extensive in space and time in a crucial altitude regime. The subcommittee judged the mission to be well conceived and the engineering to be "unusually sound" but was reluctant to recommend funding it at the expense of all other aeronomy experiments for several years. The proposal was declined.

Subsequently, the capability of the Drag-Free Satellite to perform a geodesy mission was described by Stanford in a presentation to NASA's Jerome Rosenberg and his staff from Geonautics, Inc., on 20 July 1967, and later a report [Ref. 16] was prepared giving more detailed information. In November a proposal [Ref. 17] was jointly submitted by Stanford and Professor Kaula, of the Institute of Geophysics and Planetary Physics of UCLA, to evaluate more precisely the optimum choice of orbit for a geodesy mission and to prepare a preliminary design and specification for the Drag-Free Satellite to perform it. This study is proceeding, funded through NASA's Electronics Research Center.

On several occasions beginning in 1962 we discussed applying the drag-free principle to the Navigational Satellite developed at the Johns Hopkins Applied Physics Laboratory (APL). Until recently, the satellite ephemeris prediction had been limited by uncertainties in knowledge of the earth's gravitational field. Recent improvements in data handling techniques have now so improved the gravitational model that uncertainties in surface forces (principally radiation pressure and atmospheric drag) are the limiting factors. Although always interested, this winter APL decided the drag-free principle was the next appropriate step in the development of the Navigational Satellite System. Because of our work in this area we were invited to propose a disturbance compensation system (DISCOS) which could be added to the existing satellite. The concept and preliminary details were worked out cooperatively with APL and submitted in May, 1968 [Ref. 18]. We understand that our proposal has been approved and that we will be preparing hardware for flight in 1970. This experience will be invaluable to us in our preparation for the General Relativity experiment. The flights will also

(1) Provide flight demonstrations of the drag-free control concept,

(2) Produce additional valuable data on the earth's gravity field, and

(3) Contribute a step toward much broader usability of navigation satellites, particularly toward their use for civil purposes.

Since its inception, this project has been principally funded by NASA under Research Grant NsG-582 with supplemental support from the U. S. Air Force under our initial contract AF33(615)-1411 and presently under contract F33615-67-C-1245. This Status Report describes the work performed under the combined funding.

II. SUMMARY OF PROGRESS DURING REPORT PERIOD

A. INSTRUMENTATION OF THE RELATIVITY SATELLITE

The analysis of the problems associated with data processing and instrumentation by Mr. Van Patten and Dr. Everitt has continued. The work in the first part of the last six-month period has yielded an important new concept which has added considerably to our confidence as to the suitability of the instrumentation system described in the Fifth and Eighth Semi-Annual Status Reports (Physics Portion). The concept is that of an inner pointing loop for the gyro-telescope assembly. It has been described in detail in the Ninth Semi-Annual Status Report (Physics Portion), May, 1968. The inner pointing loop provides a means of increasing the telescope pointing accuracy to such levels (< 0.1 arc sec) that errors due to telescope nonlinearity should be reduced to the desired level of .001 arc sec. Without the inner pointing loop it had been necessary to consider various more complex alternatives as listed in the reference, besides introducing accuracy problems such as larger noise equivalent angle and zero offset in the case of a telescope with a defocused image. The introduction of the inner servo allows us to use a telescope with full accuracy by operating at or near the diffraction limit.

Analysis of the inner pointing loop and interrelation with attitude control will be started during the next reporting period.

B. DEVELOPMENT OF A LOW-NOISE GYRO READOUT AMPLIFIER FOR THE RELATIVITY EXPERIMENT

A working model of a low-noise preamplifier has been designed and breadboarded by David de Pietro. This preamplifier coupled with a post-amplifier comprises the front end of the gyro readout electronics.

The low-noise preamplifier, having a gain of 60 db at 100 KHZ, consists of three direct-coupled nondegenerative stages followed by a double-ended differential amplifier stage with heavy degeneration. The total power dissipated by the complete preamplifier circuit is 1.5 mw.

The circuit has been deliberately designed with extremely low heat dissipation so that it can be placed within the cryogenic environment, surrounded by a superconducting magnetic shield to eliminate pickup. The circuit itself will operate at about -70° C. A special inside-out dewar box to contain the circuit is being designed by P. Worden of the Physics Department. To obtain maximum possible readout accuracy the noise figure of the amplifier must also be minimized. This is accomplished by careful front-end design, including use of a low-noise field-effect transistor, and a novel bias arrangement which allows rejection of noise power introduced by bias resistors. Calculations show that 0.5 db noise figure is attainable with this design, although so far 7 db has been measured. The lower noise figure is possible with a properly designed input transformer.

The preamplifier output signal is differential, which provides common mode rejection of noise pickup along the cable from the preamplifier to the postamplifier.

C. ATTITUDE CONTROL OF THE RELATIVITY EXPERIMENT

1. Control Concepts

In addition to being central to the success of the Relativity experiment, this attitude control problem is intriguing and challenging because it requires precisely pointing one body by pushing on another body which is only loosely coupled to the first.

The studies reported in Ref. 20 and suggestions of Dr. Chandler have led us to a tentative control configuration wherein the inner body of the system, Fig. 1, containing the telescope and gyros, is held within 0.1 arc sec of the star by fast cryogenic actuators pushing against the middle body. The middle and outer bodies are connected by light tension wire. The control of the outer body, using the helium-gas thrusters, will be of a slower, follow-up nature. Analysis and control-system synthesis of this system has been started to assure that it can provide sufficiently tight control with acceptably low weight, power and (especially) complexity.

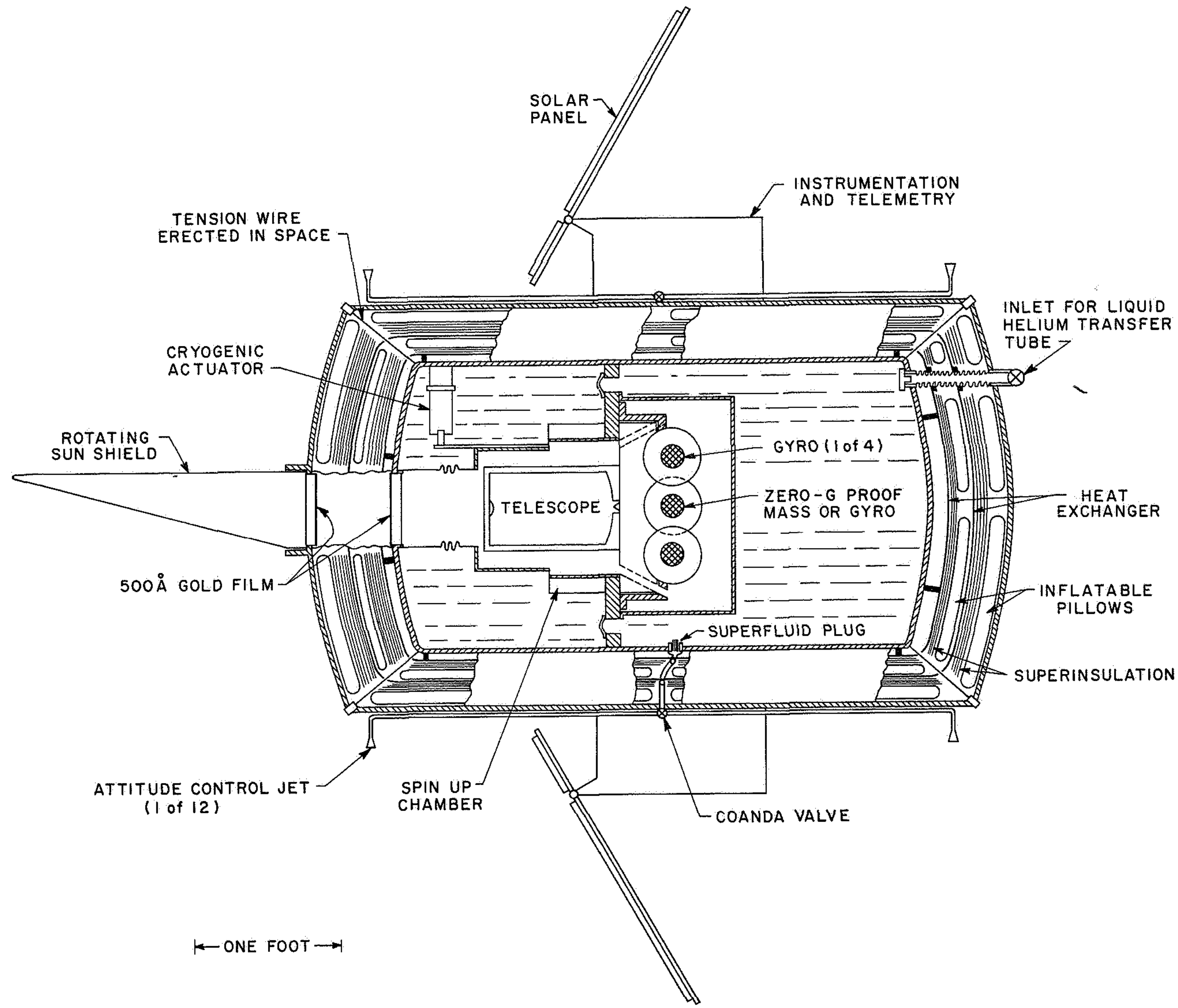


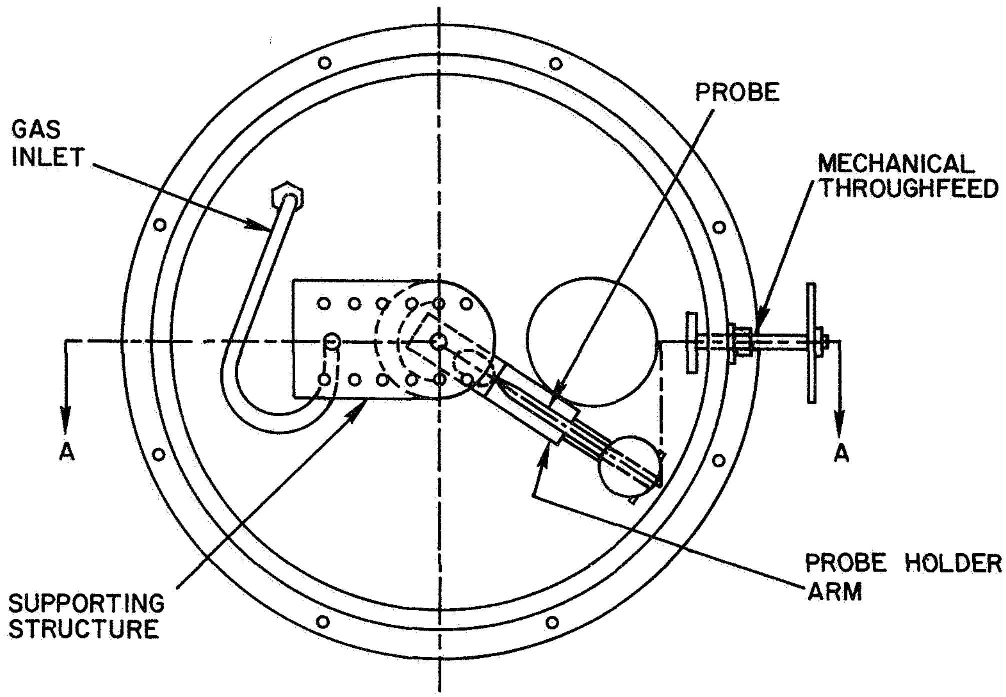
FIG. 1. GENERAL VIEW OF RELATIVITY SATELLITE

2. Helium Thruster Design

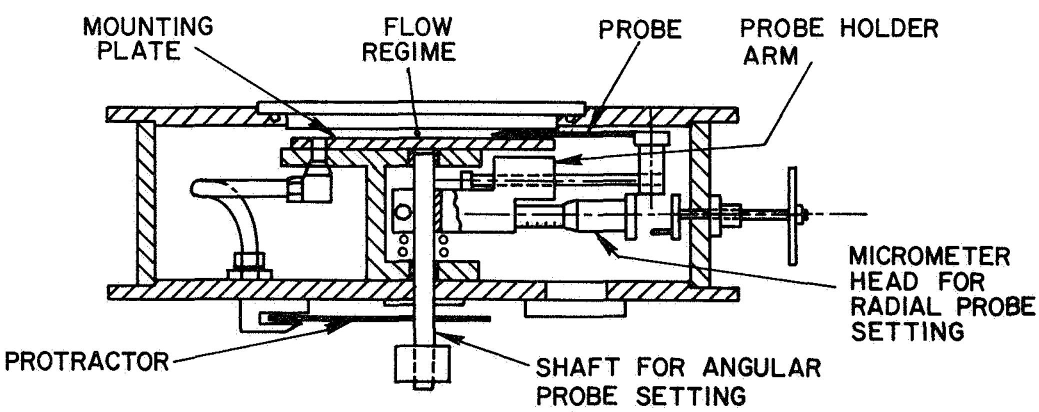
a. Basic Concept. The package for the Relativity experiment will be kept at cryogenic temperatures by carrying the whole package in a dewar filled with liquid helium. Heat leaking through the superinsulation of the dewar causes the helium to boil off at a rate of approximately 1.2 mg/sec. The helium gas thus produced can be used in a cold gas thruster system to control the attitude and translation of the satellite. This has been shown in Ref. 10, pp. 24-25.

b. Coanda Valve. A basic requirement for the control system is that it has to work for a period of at least one year without any failure whatsoever. This has led to a search for methods of modulating the helium gas flow to the individual thrusters that do not involve mechanically moving parts. Six methods have been discussed in Ref. 10, pp. 27-28. Of these, a valve utilizing the Coanda Effect (attachment of fluid jet to adjacent curved wall) has been chosen for detailed studies. It is described in Ref. 10, pp. 28-31. An experimental program has been initiated, since little is known about the effect under the conditions prevailing in the Relativity satellite (small Reynolds Number, supersonic jet), and since theoretical investigation of the problem is prohibitively difficult. During the report period, a test chamber or "variable density wind tunnel" has been built for this purpose, and preliminary measurements have been performed.

c. Test Chamber. The test chamber is a cylindrical chamber (14" diameter by 4" high internally) in which the pressure can be varied between 760 mm Hg and about 30 microns -- ultimate vacuum with no gas input (see Fig. 2). It is connected to the same Stokes 50 cfm single-stage vacuum pump that is used for the Experimental Program in Thrust Determination [Ref. 8, pp. 5-19]. Test nozzles and other test geometries are arranged in a gap 1/4" high that lies immediately below a large plexiglass window. The test geometry can be set up outside the test chamber on a mounting plate which is then bolted to a supporting structure inside the chamber. The flow field generated by the test nozzles is investigated with probes (pitot probe, temperature probe) that are mounted on a rotating arm. The shaft on which the arm is mounted goes through the



FRONT VIEW WITH FRONT PLATE AND MOUNTING PLATE REMOVED



SECTIONAL VIEW, SECTION A-A

FIG. 2. TEST CHAMBER FOR COANDA FLOW

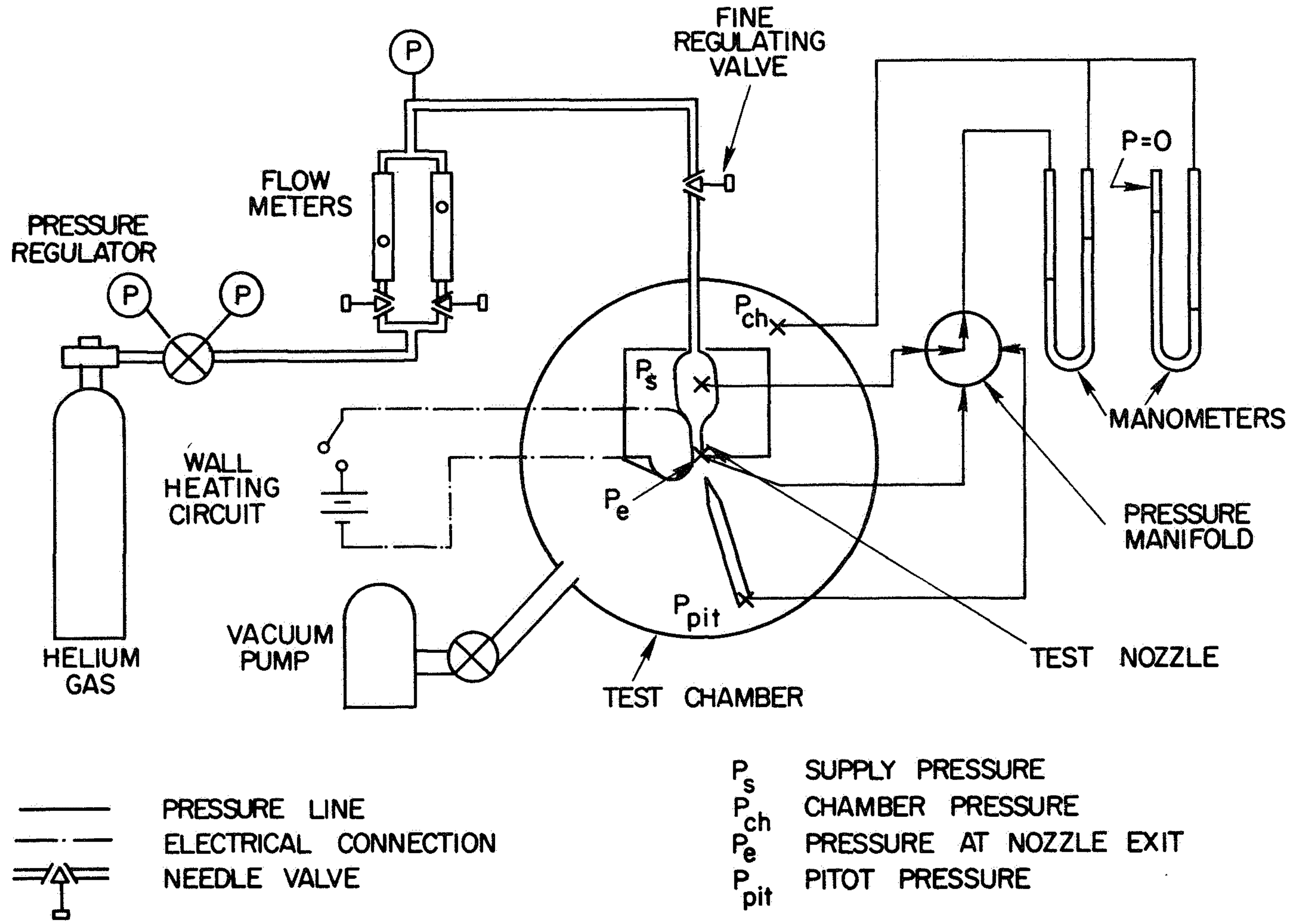


FIG. 3. INSTRUMENTATION DIAGRAM OF LOW DENSITY TEST CHAMBER

chamber wall to permit angular positioning of the probes. Also, the length of the arm (i.e., the radial position of the probes) can be adjusted from outside by a mechanical throughfeed. This method of moving the probes in polar rather than rectangular "coordinates" has been chosen because the flow field issuing from a Coanda Nozzle is essentially radial rather than parallel.

Instrumentation so far includes two "Matheson" rotameter flowmeters, and oil and mercury manometers to compare the pressure at various points in the path of the gas flow with the chamber pressure. The latter is also measured with an absolute mercury manometer to ±0.1 mm Hg. A thermocouple gauge is used to check leak tightness of the chamber. Fig. 3 shows the instrumentation diagram.

d. Test Results. So far, three types of tests have been performed:

1) General system check-out
2) Symmetrical plane helium gas jets at low Reynolds numbers and different Mach numbers
3) Coanda Effect in a steady state.

1) System Check-Out. Here are to be mentioned the calibration of the flowmeters, of the oil manometers, and of the pitot probe.

Pitot tubes give pressure readings that differ from the true stagnation pressure at probe Reynolds Numbers below 40 due to viscous effects. The error depends on probe shape, Reynolds Number, Mach Number, and, at pressures below 1 mm Hg, also on probe temperature. A rough idea of the effect has been obtained by investigating symmetrical jets with Reynolds Numbers between 170 and 27 with different pitot probe heads. First, several hypodermic needles were tried. They resulted, however, in a prohibitively slow manometer response. Next, conical probe heads with 15° cone half angle and with .015", .020" and .030" bore diameters were tried. No difference in pressure reading could be detected among them. However, large differences in response (the response time constant varies with the minus fourth power of the bore diameter) and in resolution of

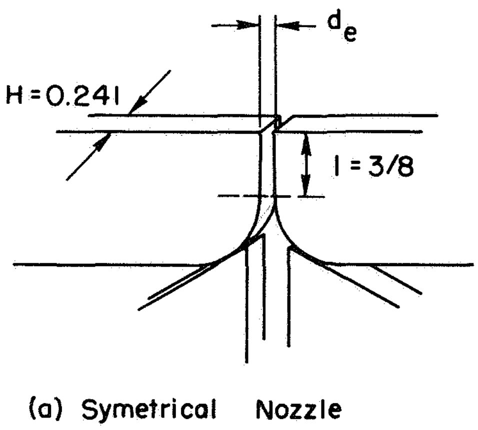


FIG. 4. RECTANGULAR NOZZLES

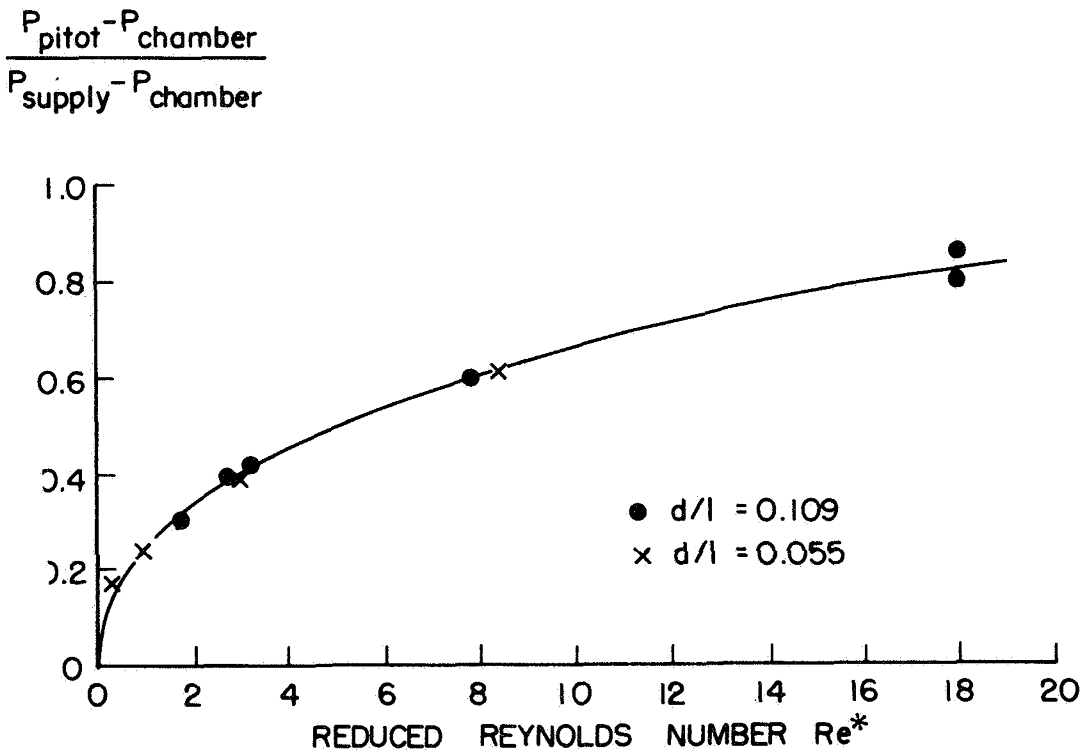


FIG. 5. RATIO OF DYNAMIC HEAD AT NOZZLE EXIT TO STATIC PRESSURE DROP ACROSS NOZZLE PLOTTED AGAINST REDUCED REYNOLDS NUMBER

the flow field were observed. To speed up measurements, the largest size head compatible with the size of the gas jets was used subsequently.

2) Symmetrical Plane Helium Jets. Measurements were performed on jets issuing from a slot-type nozzle that was formed between two plexiglass blocks [see Fig. 4a]. The slot dimensions were 1/4" high by 3/8" long while two widths, .021" and .040", were used. The following results were obtained:

- The discharge characteristics of the nozzle depend very strongly on the "reduced Reynolds Number"

$$Re^* = \frac{\zeta \bar{v} d^2}{\mu \ell} = \frac{\dot{m}}{H\mu}\frac{d}{\ell}$$

where

ζ = local gas density

$\bar{v}$ = average gas velocity in slot

μ = viscosity at supply conditions

d = slot width

ℓ = slot length

H = slot height

$\dot{m}$ = gas mass flow rate; $\dot{m} = \zeta \bar{v} H d$.

This is evident from Fig. 5 which shows the ratio of measured dynamic head at the nozzle exit to the total available pressure drop across the nozzle plotted against the reduced Reynolds Number.

- The peak dynamic pressure along the axis of subsonic jets is related to the distance x from the nozzle exit by the law

$$\frac{p(x)}{p(x=0)} = \left(\frac{1}{1 + x/x_o}\right)^{2/3}$$

where

$x_o = c_J \, d_e \, Re$;

c_J = 0.04 in the present measurements

d_e = nozzle exit width

$Re = \dot{m}/(H\mu)$ Jet Reynolds Number

This formula, except for the empirical coefficient c_J, can be deduced from the theory of laminar jets in Ref. 3, p. 168. It predicts that at a jet Reynolds Number of 10, which is about the value to be encountered in the Relativity experiment, jet momentum decays to one half of its value at the nozzle exit at a distance of only 0.74 nozzle widths downstream of the nozzle exit. This confirms the statement made in Ref. 10, p. 28, that control methods based on jet momentum (the Coanda Valve belongs to this class) are not suitable for the Relativity experiment.

- Supersonic, underexpanded jets with Reynolds Numbers above 100 exhibit the familiar expansion diamonds which are well known from high Reynolds Number flow. The wavelength of the expansion pattern allows estimating the nozzle exit Mach Number according to the formula

$$\lambda/d_e = 2\sqrt{M_e^2 - 1}$$

where

λ = wavelength of expansion pattern

d_e = nozzle exit width

M_e = nozzle exit Mach Number.

3) Coanda Effect. Preliminary tests on the Coanda Effect and its dependence on wall temperature were conducted with a nozzle as shown in Fig. 4b. So far the curved wall radius r was 5/32" in all cases. A strip of .001" gauge aluminized Mylar was used as a resistive heater to heat the curved wall. Electrical conductivity measurements indicated the aluminum layer to be about 200 Å thick. Overheating the parts of the heating strip that are not exposed to the gas flow was prevented by vacuum depositing another 500 Å of aluminum on those parts, thus locally decreasing the resistance and the power dissipation. The following results were obtained:

- In spite of the negligible thermal inertia of the resistive aluminum layer and in spite of the excellent insulation properties of plexiglass and Mylar, the time needed to reach thermal equilibrium is of the order of five minutes.

- While subsonic and slightly supersonic jets are deflected toward the curved wall (classical Coanda Effect), the deflection angle decreases as the pressure in the chamber outside the nozzle is lowered. In the tests, the jet deflection angle θ [see Ref. 10, Fig. 15, p. 29] could be varied between $+30^\circ$ (toward curved wall) and -4° (away from wall) by changing the downstream pressure while the supply pressure was kept constant.

- The change in deflection angle due to wall heating seems to be independent of the original deflection. At a jet Reynolds number of 52, a "gain" of about 2 arc degrees per Watt of heating power was measured.

e. Conclusions.

1) The Coanda Effect or any technique to modulate the gas flow which is based on jet momentum is not feasible in the Relativity experiment. An alternate method utilizing capillary restrictors has been proposed. The resistance of capillary tubes depends strongly on temperature, the dependence increasing with decreasing Reynolds number. This phenomenon can be used to make a temperature-controlled, no-moving-part valve that is particularly suitable for low Reynolds number flow.

2) The Coanda Effect could be observed, even in a broader sense than expected (jet deflection toward and away from curved wall). The change of jet deflection angle with wall temperature or, rather, with wall heating power, seems to be remarkably independent of pressure fluctuations. This is a very favorable point in applications in an electric-to-fluidic transducer.

3) The terminal design needs drastic improvement to make a Coanda valve feasible.

D. FIXED-BASE SIMULATION OF THE RELATIVITY EXPERIMENT

We consider it of vital importance to undertake exhaustive laboratory simulation of the experiment in the presence of vehicle dynamics, in order to evaluate the design of the attitude control and inner pointing loop, and to explore the effects of residual attitude motions on the performance of the readout systems. The completion of the designs of hardware for the first laboratory model now makes it important to address the entire simulation problem as soon as possible.

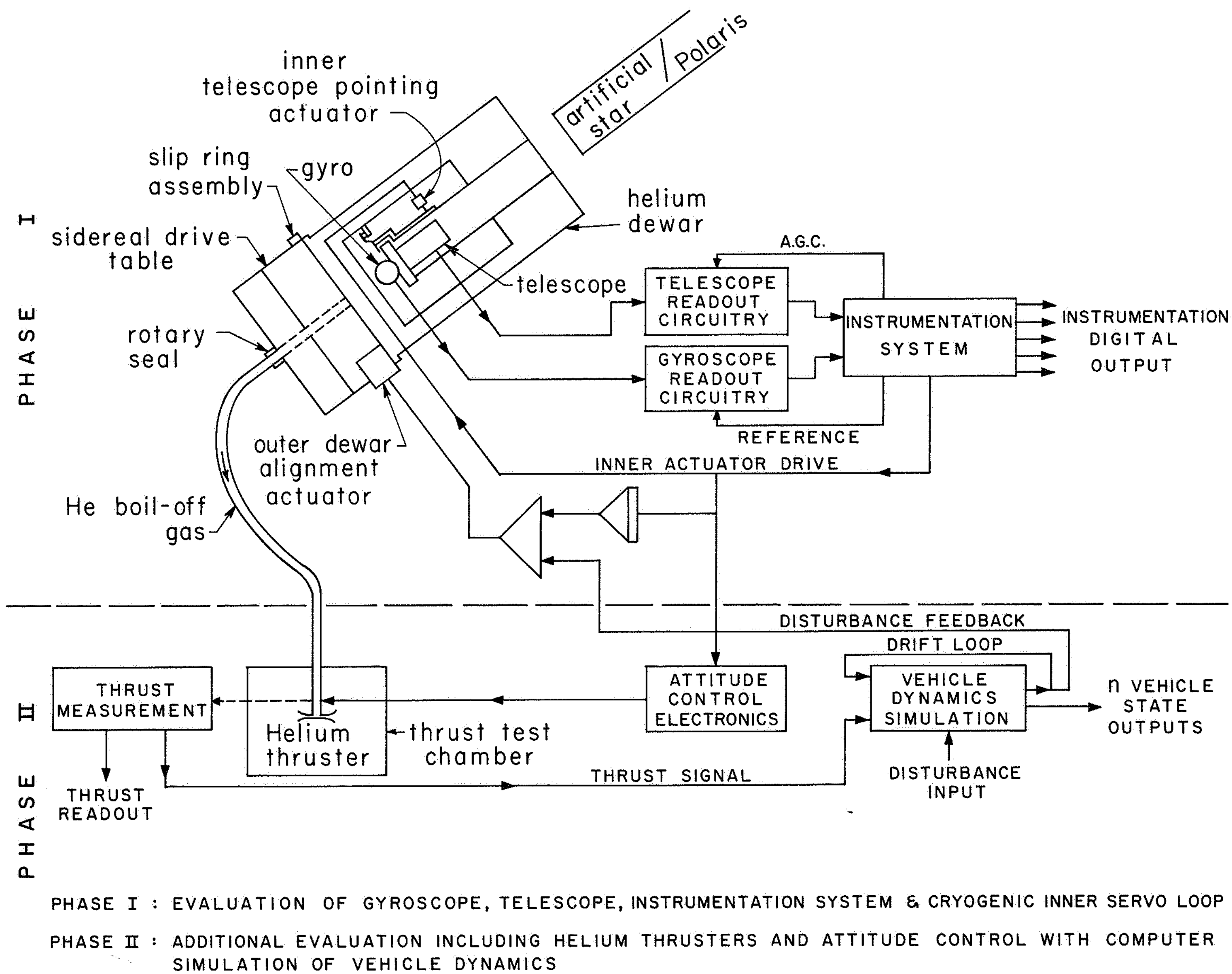


FIG. 6. PROPOSED FIXED BASE SIMULATION OF RELATIVITY EXPERIMENT (one axis shown)

The arrangement for fixed-base simulation is shown in Fig. 6. The inner part of the flight experiment (Fig. 1) is represented by a telescope and one gyro suspended electrostatically to one g. This inner part will be suspended in a liquid helium dewar, and will be rotatable, relative to the dewar, by means of cyrogenic actuators, just as in the flight vehicle. The telescope can look through the laboratory roof at Polaris.

The dewar itself will not be movable (as in flight) but will be fixed rigidly to the sidereal drive. The dynamics of dewar motion will be simulated electrically in a computer. Actual helium-gas thrusters will be used in the simulation. They will use boil-off gas from the dewar, and will operate in a vacuum chamber, their thrust being reported to the computer that is simulating vehicle dynamics

Detailed design of the simulation is proceeding in parallel with the control system studies described above (Section C).

E. DRAG-FREE SATELLITE CONTROL SYSTEM SIMULATION

In performing fuel consumption and control effectiveness tests on the rotating Drag-Free Satellite simulator [Refs. 9 and 10], a certain phenomenon which we call "trapping" has been encountered. The phenomenon was identified with the aid of state estimation techniques implemented on our TR-20 analog computer adjacent to the satellite simulator.

The existing control mechanization for the simulator consists of pulse-width pulse-frequency modulators with deadbands and lead compensation in each axis and the rotational cross-coupling terms. The deadbands along each body-fixed axis create a square deadspace in a plane. In Fig. 7, lines of control direction are sketched in the first quadrant around the deadspace. Note that in the vicinity of the deadspace corner, the force is in the wrong direction except on the 45° line (it should point toward the origin). This fact, coupled with small errors in deadspace squareness, cross-coupling terms, or mass-center alignment, gives rise to control-force directions sufficiently erroneous to cause the proof mass to become trapped near a deadspace corner. Because the satellite is spinning, the mass center (origin in Fig. 7) is describing circles in

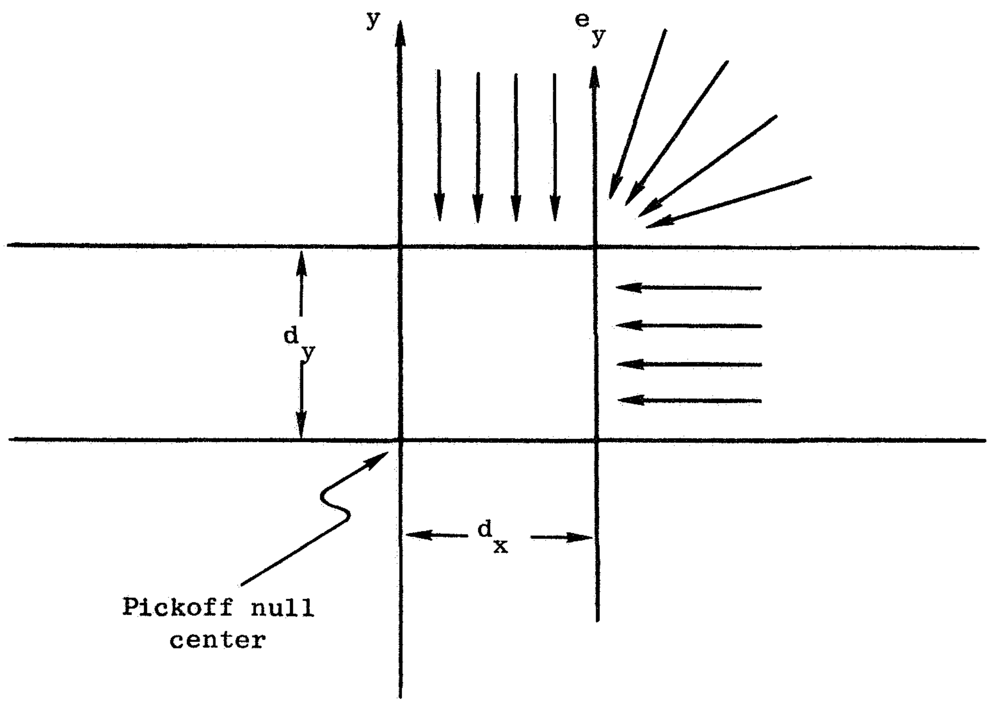


FIG. 7. PLANAR DEADSPACE

inertial space. This state is stable and the control force continues with just the right direction and magnitude to balance the centrifugal force of the mass center's circular motion, thus wasting propellant. In the simulation, small errors (20 percent) in mechanizing the deadspace were sufficient to cause the vehicle to become trapped consistently even with perfect alignment of the center of mass (c.m.) and the sensor null. This information was provided by the TR-20 which yielded estimates of the c.m. accurate to within .001". (See Section F below.)

Investigation of the system coupled with the TR-20 estimator is continuing to verify our analysis of the phenomenon. This information will now be used to develop a suitable replacement design.

F. DRAG-FREE CONTROL SYSTEM ANALYSIS: STATE ESTIMATION TECHNIQUES

If a dynamical system is thought to obey the set of system equations

$$\dot{x} = Fx + Gu$$

where

$$x \triangleq \text{state vector},$$

$$\left.\begin{matrix}F\\G\end{matrix}\right\} \triangleq \text{constant matrices},$$

$$u \triangleq \text{known control vector},$$

an "estimator" or "observer" for this system is

$$\dot{\hat{x}} = F\hat{x} + Gu + K(y - H\hat{x})$$

where

$$\hat{x} = \text{estimated state vector}$$

$$K = \text{constant estimator gain matrix}$$

$$y = \text{measurement vector}$$

$$H = \text{constant matrix.}$$

Recent references covering the theory of these state estimation techniques are Refs. 21 and 22.

The normalized translational equations of planar motion of a rotating body in body coordinates may be written

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{v}_x \\ \dot{v}_y \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \omega^2 & 0 & 0 & +2\omega \\ 0 & \omega^2 & -2\omega & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ v_x \\ v_y \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} f_x^c \\ f_y^c \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ f_x^d - \omega^2 X_e \\ f_y^d - \omega^2 Y_e \end{bmatrix}$$

where

x, y = position of mass center with respect to inertial space,

v_x, v_y = time rate of change of position,

f_x^c, f_y^c = control accelerations,

ω = spin rate,

f_x^d, f_y^d = disturbance accelerations,

X_e, Y_e = center of mass displacement.

If we assume

(1) disturbances and c.m. displacement negligible $(f_x^d - \omega^2 x_e = 0,$ etc.)

(2) x,y are measured directly,

then one of the most simple examples of an estimator is:

$$\begin{bmatrix} \dot{\hat{x}} \\ \dot{\hat{y}} \\ \dot{\hat{v}}_x \\ \dot{\hat{v}}_y \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \omega^2 & 0 & 0 & 2\omega \\ 0 & \omega^2 & -2\omega & 0 \end{bmatrix} \begin{bmatrix} \hat{x} \\ \hat{y} \\ \hat{v}_x \\ \hat{v}_y \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ f_x^c \\ f_y^c \end{bmatrix} + \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \\ K_{31} & K_{32} \\ K_{41} & K_{42} \end{bmatrix} \begin{bmatrix} x - \hat{x} \\ y - \hat{y} \end{bmatrix}.$$

If f^d and c.m. displacement are not negligible, errors will be present in the estimate of the states (x and v). If these errors are not acceptable or if we desire knowledge of the unknown quantities, then the state vector is augmented with these errors as constants to be estimated. The estimator equations then become:

$$\begin{bmatrix} \dot{\hat{x}} \\ \dot{\hat{y}} \\ \dot{\hat{v}}_x \\ \dot{\hat{v}}_y \\ \dot{\hat{x}}_e \\ \dot{\hat{y}}_e \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ \omega^2 & 0 & 0 & 2\omega & -\omega^2 & 0 \\ 0 & \omega^2 & -2\omega & 0 & 0 & -\omega^2 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{x} \\ \hat{y} \\ \hat{v}_x \\ \hat{v}_y \\ \hat{x}_e \\ \hat{y}_e \end{bmatrix} + \begin{bmatrix} 00 \\ 00 \\ f_x^c \\ f_y^c \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \\ \\ K \\ \\ \\ \end{bmatrix} \begin{bmatrix} x - \hat{x} \\ x - \hat{y} \end{bmatrix}$$

for negligible disturbance forces but with c.m. being estimated. This system was mechanized on the TR-20 as described in Section E.

G. EXPERIMENTAL INVESTIGATION OF THRUST IMPULSE

The effort during this period involved completion of design and fabrication of the valve and completion of the pressure measurement instrumentation to be employed in the initial tests. Part of the valve body and part of the valve poppet are shown in Fig. 8. Two pressure measurements are to be made in the chamber upstream of the nozzle -- one

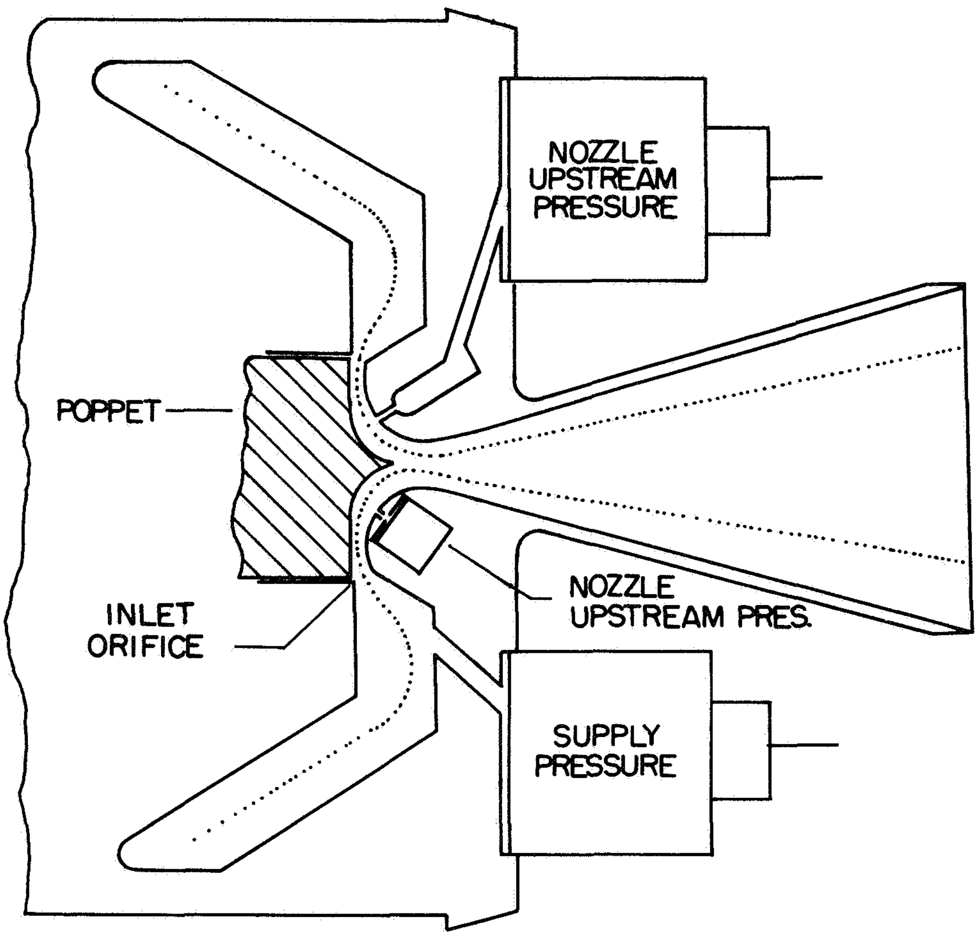


FIG. 8. VALVE CONFIGURATION FOR INVESTIGATING PRESSURE TECHNIQUES FOR FORCE MEASUREMENT

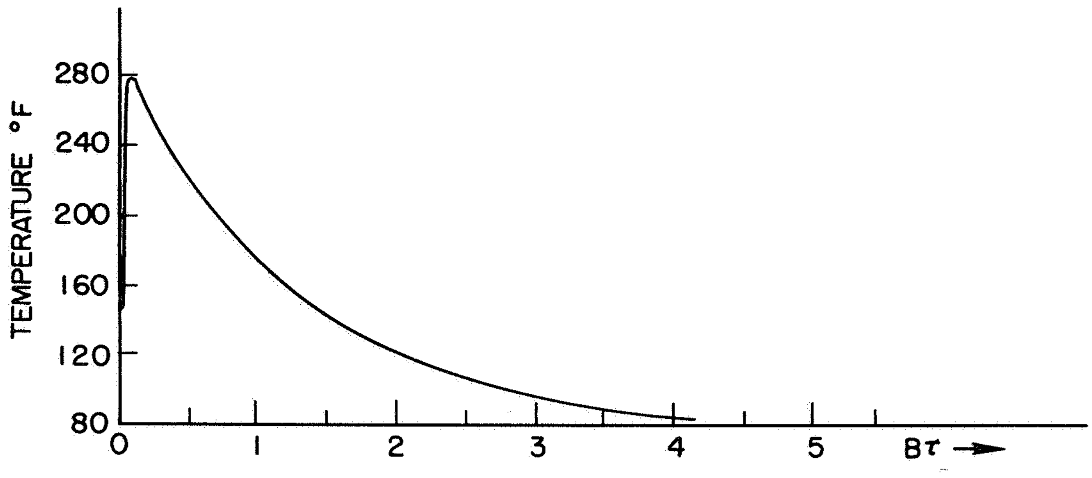


FIG. 9. TEMPERATURE HISTORY DURING BUILDUP

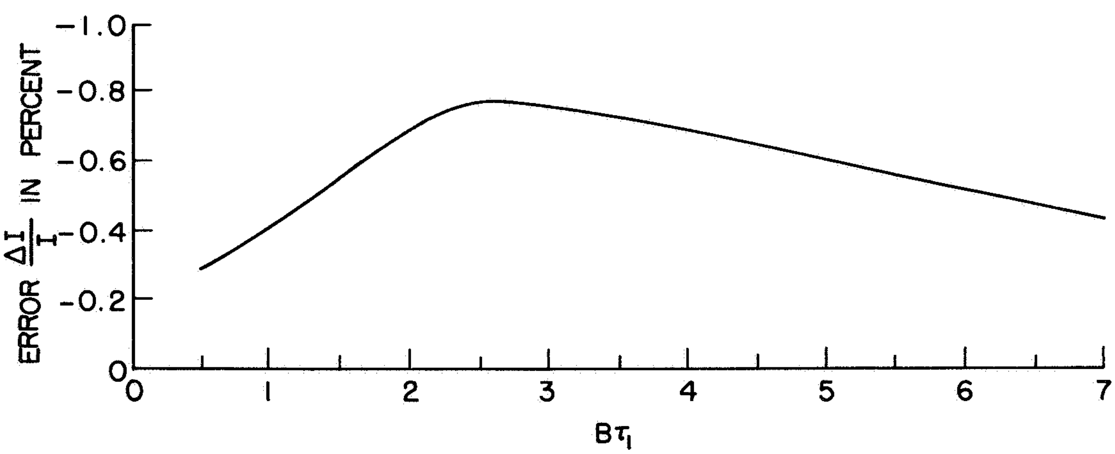


FIG. 10. PERCENT ERROR IN MEASURED IMPULSE

with a small fast responding transducer, the other with a larger and more accurate but slower responding transducer system. The supply pressure upstream of the inlet orifice is also measured.

A simplified analysis has been made of the simplest mode of measuring thrust impulse. In this method, the inlet orifice shown in Fig. 8 is made a sonic orifice, and the measurement of thrust impulse is made by integrating the supply pressure for the period of time the valve poppet is open (valve "open time" technique of measurement). The results of this analysis are given in Figs. 9 and 10.

Assuming instantaneous opening of valve poppet, Fig. 9 shows the temperature of the gas in the chamber upstream of the nozzle. An approximate expression for the equation for this temperature is

$$T = T_o \left[1 + \frac{2}{3} \frac{e^{-B\tau} \left[1 - e^{-\frac{3}{2}(\gamma - 1)B\tau} \right]}{(1 - e^{-B\tau})} \right]$$

T_o = reservoir temperature

$B\tau$ = nondimensional time

$B\tau = 3$ corresponds to one millisecond for the valve in question.

The temperature after closure of the valve is

$$T = T_1 \times \frac{1}{\left[\frac{\gamma - 1}{2} B(\tau - \tau_1) + 1 \right]^2}$$

T_1 = temperature at instant of valve closure.

The difference in temperature history during buildup and decay of this hypothetical valve would lead one to suspect that the higher temperature during pressure buildup would compensate for the lower temperatures during decay. Such is the case as shown in Fig. 10. The increase in specific impulse due to higher temperatures during buildup in thrust almost compensates for the decrease in specific impulse that occurs due to the reduction in temperature due to the decay transient. Fig. 10 shows the percentage error in percent of impulse as a function of the pulse length. If the model is a reasonable representation of the actual valve, Fig. 10 shows that differences in buildup and decay transients may not be a serious problem when using the "valve open time" technique for measuring thrust impulse. The thrust measurement technique that appears most suitable for application to (simple and reliable) spacecraft thrusters is the "valve open time" technique, and establishing the limits of the measurement performance obtainable from this approach will be of major interest in this experimental work.

III. PLANS FOR THE IMMEDIATE FUTURE

Further study of the interaction between the Relativity experiment instrumentation system and the inner pointing loop and attitude control system will take place during the next period. A review of the electronic accuracy requirements and how they relate to the present state-of-the-art will be conducted.

Work will be started on a wide-dynamic-range, nonsaturating post-amplifier for application with the relativity gyro readout. This amplifier is needed in conjunction with the low-noise preamplifier.

We plan the following helium thruster research in the coming six months:

(1) Alternate valving techniques for use in the Relativity experiment will be studied.

(2) Gasdynamic investigation of single, supersonic and subsonic Coanda Nozzles will continue.

(3) The behavior of supersonic nozzles and jets at low Reynolds Numbers will be studied.

(4) The transient thermal behavior of a heated wall will be studied, the goal being a much improved thermal design of the Coanda nozzle with a thermal frequency bandwidth of 0 to 50 Hz.

(5) Two Coanda jets will be combined in a symmetrical fashion as outlined in Ref. 10, p. 31. The combined jet will be studied in an unconfined, open-exhaust regime, as well as in a closed interaction chamber with exit ports, loads on the ports, and vents. The goal is demonstration of the basic feasibility of an analog electric-to-fludic transducer utilizing the Coanda Effect.

We plan during the coming six months to begin a full dynamic analysis of the system for controlling the attitude of the relativity experiment, with the object of first establishing in a preliminary way the values of control parameters and physical parameters that we shall want in the flight vehicle, and second providing the basis for design of the fixed-base simulation described in Section II D (pp. 15-17).

Analysis and design of a drag-free-satellite translational control system which will be less susceptible to the trapping phenomenon is currently in progress. A system will be built on the simulator to verify the design.

Analysis is also continuing to determine the feasibility of building an estimator in the laboratory simulator. Estimation of the center of mass in this control system allows the control to center about it and eliminates need for accurate alignment of the sensor null point and the center of mass.

The data processing problems associated with output measurement data from a system such as the drag-free satellite will involve at least small nonlinearities. In order to determine the effectiveness of several standard algorithms which may be used to do this processing, we will attempt to get analytic expressions for the variances and means of the errors in those algorithms in the presence of assumed forms in those nonlinearities. Along with this, we will try to derive a simple justification for the application of these algorithms to nonlinear problems. Success in these two endeavors might enhance the understanding of nonlinear filtering problems.

In the experiments on means for measuring the thrust of a drag-free satellite, we expect in the next six months to complete assembly of the thrust stand and test servo. Concurrently, we will continue to develop the analytical model for flow in the valve and nozzle.

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